M208 Guide

Contents

Introduction

Welcome to M208! We hope that you will enjoy studying this module.

M208 introduces a wide range of pure mathematics, covering topics in **linear algebra**, **group theory** and **analysis**. The module emphasises the logical structure of mathematical arguments; proofs are constructed starting from precise definitions of mathematical objects. You are encouraged to critically analyse such proofs to gain a deeper understanding of their construction.

Algebraic pure mathematics, including linear algebra and group theory, often concerns general structures with certain properties that can unify different areas of study. We generally start by looking at specific familiar examples, such as sets of numbers, lines and planes, before abstracting certain concepts and concentrating on the properties of the structure rather than the nature of the specific elements.

Real analysis, the area of analysis covered by this module, involves the study of real functions and includes the foundations of calculus: it concentrates on rigorously establishing the validity of the mathematics that underpins calculus.

In completing this module you will come to appreciate the need for precise notation and careful argument – and that the final answer is only one part of solving a problem. You will see that quite different ideas can be interrelated by underlying concepts. You will meet new concepts and consolidate many of the mathematical ideas and methods that you have learned in earlier modules. Furthermore, successful study of M208 will set you in good stead for tackling any level three module in pure mathematics. Alternatively, if your interests lie primarily in applied mathematics, science, engineering or technology, M208 will provide you with many mathematical skills relevant to your main area of interest.

Getting started and use of technology

The background knowledge that is assumed can be obtained by successfully completing MST124 and MST125.

This means that you are expected to be familiar with standard mathematical notation and elementary functions (such as sine, cosine, logarithms and exponentials). We also assume that you are familiar with algebra, basic calculus, complex numbers and vectors. If you have not already completed the M208 Diagnostic Quiz, then you are advised to do so: it will highlight to you any areas of the assumed background knowledge that you might need to study more carefully in preparation for M208.

Most of the module materials are printed (as described below in the *Module components* section), but note that you will need your computer and internet connection in order to:

- access the study planner, which describes the activities that you should be doing each week
- download assignments
- access supplementary teaching materials (such as screencasts) made available online
- engage with fellow students and the M208 team via the module forums.

Module components

Module texts

The module is based on printed text, supplied as six books, each comprising four units of study.

In addition to a broad introductory book, there is one book on **linear algebra**, two on **group theory** and two on **analysis**.

Book A – Introduction

The first book covers the main concepts that underlie the topics in the other books. It reviews some of the important foundations of pure mathematics and the mathematical language used to describe them. Much of the material in this book will not be new to you; however, you will meet new notation and additional concepts, and learn to express your mathematics more precisely.

The ideas covered in this book will be in constant use throughout the module.

Book B – Group theory 1

The second book introduces the foundations of group theory. The word *group* describes a particular type of mathematical structure that occurs naturally in many branches of mathematics, as well as in other disciplines such as chemistry and physics.

This first book of group theory introduces the basic ideas leading up to a simple but powerful result known as Lagrange's Theorem, which underpins much of the development of the subject. The book starts by considering the symmetry of two- and three-dimensional shapes and how these ideas can be expressed mathematically.

Book C – Linear algebra

The third book sets the foundations of linear algebra. It starts by considering linear equations, vectors and matrices before introducing the concept of a *vector space*, a mathematical structure that is one of the most important unifying concepts of pure mathematics.

The elements of a vector space may be called vectors, but they can be very general objects, for example, complex numbers, functions or matrices. Vector spaces and their associated linear transformations form a very general context in which many seemingly unrelated problems can be studied using similar techniques.

Book D – Analysis 1

The fourth book introduces the study of real analysis. It discusses in detail what it means for a function from $\mathbb{R}$ to $\mathbb{R}$ to be 'well-behaved' by dealing in a precise, quantitative way

with the concept of a limit, and with the related ideas of infinite sums, and continuous functions.

There are various approaches to analysis and this module uses *sequences* as the basic tool.

Book E – Group theory 2

The fifth book builds on the material introduced in the second book (*Group theory 1*) and covers some more advanced group theory. It shows how group theory can reveal links and similarities in concepts that seem unrelated, giving a greater understanding of these concepts. It also includes examples illustrating how group theory can simplify problems that at first sight appear prohibitively complicated, making it possible to solve them.

Book F – Analysis 2

The final book builds on the concepts introduced in the fourth book (*Analysis 1*) and takes the study of analysis further. Using the ideas of limits, infinite sums and continuous functions, it puts the foundations of calculus on a firm and logical basis.

You may notice that the module texts are fairly long! This is because they provide detailed explanation of the topics covered with numerous figures and examples to help your understanding. You will also find a large number of worked exercises and exercises. The worked exercises provide examples of model solutions to help you structure your own solutions to exercises. You should attempt as many exercises as you have time for.

In the books, coloured backgrounds are used to indicate certain types of text: important results (green background), worked exercise solutions (beige background) and historical notes and interest pieces (blue background).

Some of the worked exercises and proofs contain lines of blue text, marked with the icons   . This text tells you what someone doing the mathematics might be thinking, but would not write down, or what a lecturer might say to explain the thinking behind the mathematics, but would not write on the board. For a worked exercise it should help you understand how you might approach a similar exercise yourself while giving a full, correctly structured solution. For a proof it should make the ideas of the argument easier to follow.

Proofs

Proofs are a vital part of pure mathematics: through proofs we ensure that the mathematics that we use is valid.

In this module you will see many proofs. These proofs are important – proofs are an essential part of mathematics. If you take the time to read and understand them, then they will often improve and develop your understanding of the theory, and they will also help you to develop your skill with mathematics in general and learn how to write your own proofs, which you are asked to do in some exercises.

However, some proofs can be difficult and time-consuming to read. Also, sometimes a proof may not contribute significantly to your understanding of the theory: for example, it might mostly depend on ideas that are not closely connected to the mathematics that you are currently studying, or it might consist of a largely technical and not very enlightening

check through various possible cases. It may be better for you to skip such proofs, at least initially, especially if you are short of time or if you do not plan to go on to study more pure mathematics after M208.

Throughout the module, the unit texts provide guidance about some proofs that you might choose to skip or delay reading for these reasons. You will be able to complete the module assessment without reading these proofs, but you should still try to read them, or at least follow what the proof is trying to do, if possible.

Some other proofs (and other types of module material) appear with guidance saying that the material is optional. Historical interest pieces, on a blue background, are also all optional. This material is included for your interest, and omitting it should not affect your understanding of the module.

If you intend to go on to study pure mathematics at level three, you should try to study as many of the proofs in the module as possible. For a deeper understanding of the material, you can return to these proofs after the end of the module. In fact, revisiting proofs from the module is good preparation for level three pure mathematics modules.

References to theorems and other results

All results (theorems, propositions, lemmas and corollaries) are numbered together in order by book, across the four units that make up each book. Thus each has a unique reference number: for example Theorem A1 is the first result in Book A, Lemma B42 is the forty-second result in Book B, and so on. Likewise the strategies are numbered in order by book.

Worked exercises (exercises with a model solution in place with a beige background) and Exercises (with a solution at the end of the unit) are numbered separately, in order by book. There are also online *additional exercises booklets* for each book, with these exercises also numbered in order by book. Therefore Exercise C34, Worked Exercise C34 and Additional Exercise C34 are not the same. You may find it helpful to use a suitable abbreviation of each when you are referring to them. For example, you could refer to the exercises above as Ex C34, WEx C34 and XEx C34 (where the X indicates additional or ‘extra’ exercises).

The solutions to worked exercises and exercises provide examples of the level of detail required to answer similar assignment questions, and when and how to refer to results used. For example, in the analysis units there are many versions of the Combination Rules, each identified by its title; however, when using such results it is sufficient to simply specify the ‘Combination Rules’ since the specific version should be clear from the context. In addition, for named results, such as the Combination Rules, you do not need to refer to the theorem number as well as the name of the result.

Handbook

The Handbook should be your constant study companion. It summarises the main definitions, results, strategies and methods from the module. Definitions are given in bold. It is a good idea to start using the Handbook from the beginning of your studies, so that you familiarise yourself with its contents.

You can and should take the printed copy of the Handbook into the examination with you (but no other module materials). This version of the Handbook must be the printed copy that was sent to you, rather than a version that you have downloaded and printed yourself.

The copy of the Handbook that you take into the examination is allowed to have 'basic annotation', which is specified by University regulations as follows.

> The text as printed may be supplemented by handwritten highlights (for example by a highlighter pen or by ringing, underlining or sidelining), and by corrected typographical errors. The addition of comments, marginal notes, notes in the blank spaces at the end of paragraphs and pages or on flyleaves is not permitted.

Website

It is important that you check the module website frequently for news about the module. There you will also find electronic versions of the module texts and Handbook, as well as the study planner, the tutor-marked assignments (TMAs), and specimen exam papers and solutions. Any errata will also be posted there.

The module website will give you access to the forums, which you can use to communicate with other students and with the M208 team.

The study planner is your key to understanding what tasks to do each week while studying M208. It contains links to screencasts and other study material.

Screencasts

The study planner on the module website contains links to video clips, called screencasts, that can be viewed online or downloaded for later study. Screencasts amplify ideas contained in the study text, for example by giving a real-time solution to a worked exercise.

These screencasts provide no new teaching material; they simply give the same examples in a different medium, which you might find helpful depending on your study preferences.

The study planner indicates which screencasts relate to which units.

Additional exercises booklets

Each study unit also has an additional exercises booklet, which contains additional questions and their solutions, for you to use to consolidate your learning. Usually the most effective time to try the questions in an additional exercises booklet is as you come towards the end of studying the corresponding unit. The additional exercises booklets also provide a good way for you to revise topics in preparation for the examination. They are available from the module website: by unit from the study planner in the week you study the unit, and by book in the resources section for ease of use during your revision (both versions contain the same set of exercises).

Challenging exercises

Some exercises are marked as 'challenging'. These exercises are **optional** and provide extension work for students relishing a challenge! Some are not much more difficult than

the standard exercises, while others are more challenging. You should attempt these questions only if you have time to do so.

If you intend to go on to study pure mathematics at level three, it is a good idea to attempt as many of these questions as possible as they will be good practice for your future studies – but you can do this after you have finished the module!

Assessment

The assessment that counts towards your final grade for M208 consists of seven tutor-marked assignments (TMAs) and a three-hour examination.

Your overall score

Your overall score for the module will be calculated as:

> 0.2 × your continuous assessment score + 0.8 × your examination score.

The seven TMAs count approximately equally towards your continuous assessment score.

The precise contribution of each assessment score to the overall score is given in the table below.

Assessment	Books covered	Contribution to overall score
TMA 01	A	2.84%
TMA 02	B	2.86%
TMA 03	C	2.86%
TMA 04	D	2.86%
TMA 05	E	2.86%
TMA 06	F	2.86%
TMA 07	All books (revision TMA)	2.86%
Examination	All books	80%

To pass the module you will usually need to achieve an overall score of at least 40%, and in addition achieve at least 35% in the examination.

Your grade

Based on your overall score (and on whether you have achieved at least 35% in the examination) you will be awarded one of the following results:

- Pass 1 / Distinction
- Pass 2
- Pass 3
- Pass 4 / Pass
- Fail

The Assessment Handbook, which you can also access from StudentHome, gives full details of the module results process (in Section 4).

Continuous assessment

There are seven TMAs: one TMA for each of the six books and a final revision TMA. Together they count for 20% of your overall score.

You are advised to attempt each TMA question soon after studying the relevant unit. When your marked work on a TMA is returned to you, you should carefully read all your tutor's comments, as this feedback will help you to improve your performance in subsequent TMAs.

The ability to communicate technical ideas is a key skill that is highly sought after by employers. To foster your development of this skill 5% of the marks for each TMA (apart from in TMA 01 and TMA 07) are set aside for good mathematical communication (GMC), that is, effective communication and the overall style of your answers. Your GMC mark for a TMA will be judged by your tutor and included as the mark for the 'final question' in the TMA; this question is not a real question but simply a placeholder for the GMC mark. Although there are no GMC marks for TMA 01 your tutor will comment on your communication and style in this assignment to help you improve these for the later TMAs.

To achieve a good mark for GMC, you should write your work in a good mathematical style, as demonstrated by the exercise and worked exercise solutions in the study texts. You should explain your solutions carefully, in such a way that at a later date you will be able to read and follow what you have written. You should use appropriate notation and terminology, define any symbols that you introduce, and write in proper sentences.

The TMAs also contain some questions that are designed to assess other useful transferable skills developed in M208. Some of these questions ask you to critically analyse arguments and explain errors or problems, while others ask you to write short proofs.

You can submit your TMA solutions either by post or online. If you submit online, then you must submit your solutions as a single PDF file:a Word document or a set of several PDF files will not be accepted. Please read the instructions on the module website carefully before submitting your work.

Examination

There is a three-hour examination at the end of the module. It counts for 80% of your overall score.

The M208 specimen examination paper shows the format that your actual examination paper will have, and the style and standard of the sorts of questions that it will contain. The solutions to the specimen paper show the sorts of solutions that will achieve full marks.

To help you prepare for the examination, you should go through the M208 specimen examination paper, and you will also find it helpful to go through M208 examination papers from previous presentations. However, note that M208 was refreshed for the presentation starting in October 2018 and there were some changes in approach, content and notation, so the questions in papers dated June 2018 or earlier are likely to have a few minor differences compared to those in papers dated June 2019 or later. The format of the examination was also changed slightly at the same time, as follows. Previously there were 12 Part 1 questions, whereas now there are ten Part 1 questions; this change was made to allow more thinking time in the examination, to prevent examinees from being rushed. In addition, previously in Part 2 two questions were chosen from five,

whereas now Part 2 has been replaced by Parts 2 and 3 with one question to be chosen from each of these two parts.

You can and should take your printed copy of the M208 Handbook into the examination, but no other module materials are allowed. Only ‘basic annotation’ of the Handbook is permitted, which is specified by University regulations as follows:

> The text as printed may be supplemented by handwritten highlights (for example by a highlighter pen or by ringing, underlining or sidelining), and by corrected typographical errors. The addition of comments, marginal notes, notes in the blank spaces at the end of paragraphs and pages or on flyleaves is not permitted.

Calculators are not allowed (and not needed) in the examination.

Support for your studies

It is important that you try to keep to schedule. Each assignment cut-off date is soon after the end of the last study week for the relevant units. We recommend that you try to finish the assignment questions relating to each part of a unit as soon as you finish that part. This will ensure that you stay on top of the material, and do not have to rush shortly before the cut-off date.

Your tutor

Your tutor is there to help you to understand the ideas in M208, and they will provide comments and feedback on your written assignments. You are advised to go through each marked assignment in detail, and to take note of the comments written by your tutor; they will help you to avoid similar errors in later TMAs and in the exam and help you to improve your mathematical communication. Try to attend tutorials – either face-to-face or online – where you will have the opportunity to talk to your tutor and other students directly.

Your fellow students

One of the best ways of learning is by talking about your work with fellow students. A convenient way to keep in touch with other students is to use the M208 forums, which are moderated by an experienced tutor. Face-to-face tutorials also provide a great opportunity to get together with other students.

Other support

You are not expected to study alone. Support is available from your tutor, through tutorials and through the module website. If you are struggling with the content of M208, please use the M208 forums and your tutor for support. If you experience difficulties that are not directly related to the content of M208, then you are welcome to contact your Student Support Team (see your StudentHome page for details).

Accessibility guide

This section is primarily aimed at those who may have difficulties with one or more elements of M208 because of a disability, for example.

Mathematics is a visual subject involving the use of mathematical notation, graphs and diagrams. General accessibility advice for all mathematics modules is available in the document 'Accessibility for Mathematics and Statistics modules', which you can find in the Accessibility section of the M208 website. You are advised to read this document carefully. It outlines the general accessibility options for module materials and websites, and contains advice on presenting your mathematics and completing the assessment. The Accessibility section of the M208 website also contains other accessibility resources related to M208.

Although The Open University has tried to avoid using inaccessible resources in M208, and to provide accessible alternatives where possible, some material that is core for M208 may not be easily accessible, even if you use assistive technology. You may need a non-medical helper to assist you.

If you think that you may need additional support during your study of M208 and you have not already contacted the University about this, please visit the Disability Support website. It describes the range of support services that are available, and guides you through the procedure to request extra help. Alternatively you can discuss your needs with an adviser from your Student Support Team. You can find the contact details on StudentHome.

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